UNIT 3

# Sets

* Objects in a set are called *members* or *elements* of the set.
* List notation/The roster method is pretty simple, just write the set out with all the elements e.g. {1,2,3,4}
* Sets can be **finite** {1,2,3} or **infinite** {1,2,3,…}
* We can also give sets names, like in the revision e.g. A= {1,2,3,…}
* Set builder notation is the most useful, as done in revision

**SYMBOLS**

A is a subset of B, (A is contained in B)  
Every set is a subset of itself A A, B B.

**Null set** or **∅** is a subset of every set.

**∅** is a subset of every Cartesian product.

 A is called the proper subset of B if A A ⊂ B

⊆ B but B ⊇ A i.e., A ≠ B

***No set is a proper subset of itself.***

***Null set or ∅ is a proper subset of every set.***

***To be a subset, A needs to have at least one less element than B***

B ⊇ A B is the superset of A (similar to )

P(A) = {∅, {p}, {q}, {p, q}}  All the subsets of A are called the power set

If A = {1, 2, 3}   A ⊆ U

B = {2, 3, 4}  B ⊆ U

C = {3, 5, 7}  C ⊆ U

then U = {1, 2, 3, 4, 5, 7}  U ⊇ A **AND** U ⊇ B **AND** U ⊇ C

A set which contains all the elements of other given sets is called a **universal set**

Source: http://www.math-only-math.com/subset.html

Subsets

Proper Subsets

Supersets

Powersets

Universal set

Example 1

The set of

* “The set of all x’s such that x is a positive integer”
* “The set of all x’s such that x is a positive integer less than 5”
* “3 is an element of ” or “3 is a member of ”

* “2 is an element of {1,2,3,4}”

* The set of all negative integers greater than -5
* Remember that with sets, we reduce redundancies (check revision). Therefore, the set {2,3,2} = {2,3}.
* A set with no elements is an empty set, denoted by .

# How to build new sets from old ones

* The universal set (in Number theory) is the set of ALL integers
* A subset is how we show pieces from a set. E.g. {1,3,7} is the subset of {1,2,3,4,5,6,7}
* In words, “A is a subset of B if and only if every element of A is in B”

Example 1

A = {a, e, i, o, u}

B = {a, b, c, ............., z}

Therefore, A is a subset of B, but B is not a subset of A. AND

*B is also the superset of. B ⊇ A*

Example 2

A={1,3,5}

B={1,5}

C={1,3,5}

B is a proper subset of A B ⊂ A

C is a subset of A, but NOT a proper subset of A since C=A

# Ordered Pairs

* We can combine members of a set in **ordered pairs**
* Therefore the set of P{1,2,3} can be ordered as:

(1,1) (1,2) (1,3)

(2,1) (2,2) (2,3)

(3,1) (3,2) (3,3)

# Relations

* Relations are a bit more complicated, they are a set of ordered pairs on the set. Relations are sets themselves.

e.g. = {(1.2),(2,1),(3,3)} The 3 indicates how many elements are in the set

AKA 12, 21, 33

# Cartesian product

* This is the ordered set taken from the first to the second co-ordinates
* It’s like writing the ordered pair, beginning to end in order
* The Cartesian product of a set is written such that the Cartesian product of A is A x A
* Therefore, the Cartesian product:

A x A =

*Read as A cross A. This can also be done with two different sets*

* Therefore, the Cartesian product of two different sets:

A x B =

* Ordered pairs are the subset of Cartesian products, therefore:

# Set union

* Set of elements that belong to A OR B OR to both
* The set union is denoted as:

A ∪ B = {x | x ∈ A or x ∈ B}.

Let A = {1, 2} and B = {0, 1},

then A ∪ B = {0, 1, 2},

i.e. the set of those elements that belong to A or to B.

Therefore, x ∈ A ∪ B

# Set intersection

* Set of elements that belong to both A OR B at the same time
* The set intersection is denoted as:

A ∩ B = {x | x ∈ A and x ∈ B}.

Let A = {1, 2} and B = {0, 1},

then A ∩ B = {1},

i.e. the set of those elements that belong to both A and B.

Therefore, “x ∈ A ∩ B iff x ∈ A and x ∈ B”.

*Iff = if and only if*

# Set Difference

* Called compliment of B relative to A
* Is simply the elements of A that don’t belong to B
* Aso referred to as the difference between A and B

A − B = {x | x ∈ A and x ∉ B}.

Let A = {1, 2} and B = {0, 1},

then A − B = {2},

i.e. the set of those elements that belong to A but not to B.

Therefore

“x ∈ A − B iff x ∈ A and x ∉ B”.

# Set Complement

* Is the opposite of set difference
* Use the UNIVERSAL set for this one
* Is simply the elements of U that don’t belong to A
* Denoted with a single quote i.e. A’

Let U = {0,1, 2,3} and A= {0, 1},

then A’ = U-A = {2,3}

i.e. the set of those elements that belong to U but not to A.

A’ = {x | x ∈ U and x ∈ A}.

# Symmetric set difference

* Is the elements that belong to A or B but NOT both

A + B = {x | x ∈ A or x ∈ B, but not both}

Let A = {0,1,2,3} and B = {0, 1,3,4},

then A+ B = {2,4},

i.e. the set of those elements that belong to A or to B but not both.

Therefore

“x ∈ A + B iff x ∈ A and x ∈ B, but not both”.

# Set Disjointness

* Two sets are disjoint where the have no elements in common
* Therefore, A ∩ B = ∅

# The powerset

* Ƥ (A), is the set that has as its members *all* the subsets of A.
* The cardinality of Ƥ (A) is

Let B = {1, 2, 3}. Which sets are all subsets of B?

The size of B is , therefore it is 8